9 N D 8 (9 44042/p 7555 7

ON THE ARENARIUS OF ARCHIMEDES.

BY

STEPHEN PETER RIGAUD, M. A.

SAVILIAN PROFESSOR OF ASTRONOMY.

INSTITUTED MDCCCXXVIII

OXFORD,
PRINTED BY S. COLLINGWOOD, PRINTER TO THE UNIVERSITY, FOR
THE ASHMOLEAN SOCIETY.
MDCCCXXXVII.

N
III
8
19

WELLCOME
LIBRARY
INSTITUTE

ON

THE ARENARIUS OF ARCHIMEDES.

READ TO THE ASHMOLEAN SOCIETY, NOV. 11, 1836.

Of all the mathematicians of antiquity, Archimedes is confessedly the first for power and originality. Every one knows his use of hydrostatics in the examination of Hiero's crown, and the extraordinary machinery by which he enabled his countrymen to defend the city of Syracuse against the Romans: his geometrical works are efforts of transcendent skill; and in his treatise, entitled Ψαμμίτης, or Arenarius, he shews the means of accomplishing what in his time appeared to all others to be impossible. This treatise stands, from its subject, distinct from the rest of his works, but it is not on that account the less interesting or valuable. It gives us, indeed, no specimen of that beautiful geometry, in which the ancients taught by their example the most perfect form of close and logical reasoning; but if its want of this attraction has made us less familiar with it, the same circumstance adds a certain variety to the method of investigation which it pursues.

Archimedes was related to the kings of Sicily, and addresses his work to Gelo the eldest son of Hiero. It appears that the grains of sand at Syracuse had been held by some to be infinite, and that even those who could not admit such an unqualified assertion, still considered them to exceed any numbers that could be assigned for them. This seems to be a speculation from which no practical advantage was likely to be produced, and none possibly was derived from the mere resolution of the question; but in the means which Archimedes devised for this purpose, we find the principle not only invented, but brought into actual operation, which in our later times has formed one of the greatest means of shortening labour in the conduct of arithmetical processes. The excellence of the Oriental numerals has reduced the Grecian arithmetic to an object of historical curiosity, and we can only admire the ingenuity of those who could work with such awkward implements. The Arenarius, indeed, is employed rather on the arrangement than the notation of numbers, but the imperfection of that notation would, in any common hands, have probably soon put a stop to the inquiry. No one can read the treatise without finding how much more clearly he can follow the reasoning of it, by reducing the several parts to the figures which we have now in use; how much, therefore, must the difficulty have been increased when the ideas to be expressed were entirely new! Archimedes, however, confident of his powers to overcome the difficulty, at once endeavoured to take it in its greatest possible extent, and asserted that he could assign the numbers which should exceed not only the sands of Syracuse and Sicily, but of what would be sufficient to fill a sphere, equal to the earth, or even to the world itself.

Κόσμος, by most of the ancient astronomers, was understood to denote a sphere, of which the radius was equal to the distance of the sun from the earth; but Archimedes mentions that the views of Aristarchus had given it a wider sense, and he is thus incidentally led to describe one of the most precise accounts which has been preserved to us from antiquity, of the true system of the universe: *ὑποτίθεται τὰ μὲν ἀπλανῆ τῶν ἄστρων, καὶ τὸν ἅλιον, μένειν ἀκίνητα· τὰν δὲ γᾶν περιφέρεσθαι περὶ τὸν ἅλιον, κατὰ κύκλου περιφέρειαν, ὃς ἐστὶν ἐν μέσῳ τῷ δρόμῳ κείμενος.* It is then added that the fixed stars are set in a sphere concentric with the sun, and that the radius of it has the same ratio to the circle in which the earth revolves, as the surface of a sphere has to its centre. This latter part of the statement alarmed the rigid geometry of Archimedes, who points out that the proportion is thus made to belong to quantities, between which there is no analogy. This is so true, that a definite meaning can hardly be extracted, with any confidence, from the expression; and Archimedes has justly objected, that the centre, being a point, has no magnitude, and consequently can have no ratio to a surface; it may, indeed, be added, that this surface itself can have no definite magnitude independent of the radius with which it is supposed to be generated: still it must be borne in mind that we have not the original treatise of Aristarchus, the context of which might assist us in understanding the manner in which he may have intended to use this expression. It is not impossible that he may have thought the distance of the fixed stars so great, that the very circle of the earth's orbit was no more than as a central point, with regard to the spherical superficies in which they were placed. This sublime idea may appear to partake too much of modern discoveries, but it is clear that Aristarchus had advanced far beyond the views of his contemporaries; and although he still retained the common impression of the fixed stars being all at an equal distance from us, this theory is even now abandoned only from reasoning on general analogies, not from demonstration. But after all, there was nothing here for the geometer to stand upon, and such an immeasurable magnitude would indeed have made it impossible to have met the problem in the extent under which Archimedes was willing to take it; but he might have limited his conditions, and could have easily done so, without diminishing their general effect.

Archimedes, however, endeavours to extract a manageable meaning from the description of the system of Aristarchus. Unwilling to abandon the idea of the sun's motion, he says, that as the earth was generally taken for the centre of the universe, the intended ratio must have been, that its magnitude was the first of three proportionals, of which the second was the *κόσμος*, (in the common acceptation of the word,) and the third the sphere of the fixed stars. This gave him quantities which must have been considered as an ample allowance for the size of his largest space: indeed in all his calculations he uniformly endeavours to exclude all cavil and objections, by going beyond what might be necessary for establishing the truth, and in consequence is led from any attempt at minute accuracy in his fundamental quantities. For his smallest body he concludes that a sphere, one *δάκτυλος* in diameter, could not be supposed to contain more than 640 millions of grains of sand; and for his largest receptacle, that the sphere of the fixed stars might be taken with a diameter of not more than 100,000,000,000,000, or 100 million of millions of stadia. According to Arbuthnot, the στάδιον contained 9600 δάκτυλοι, but Archimedes takes it in round numbers, as not exceeding 10,000: and the contents of spheres varying as the cubes of their diameters, a general notion may be formed of the enormous extent to which the larger number must be increased, in order to specify how many grains of sand would be required to fill such a universe as is here imagined.

With all the advantages of modern arithmetic, our common enumeration would soon become burdensome in such a problem; but the difficulty of it was increased in an inconceivable degree, by the Grecian system of marking distinct numbers by the successive letters of the alphabet. With the addition of three symbols (ϛ ϟ ϡ) for 6, 90, and 900, they were thus able to denote all the digits, tens, and hundreds; a peculiar mark, not unlike an *iota subscriptum*, converted the nine first symbols into designations for so many thousands, and M stood for a myriad, the multiples of which were denoted by numeral letters which were annexed to it. Mere addition in such an arrangement required a strain of attention, but the trouble and tediousness of the work were still greater in multiplication. This labour was even further increased by their practice of working from left to right, beginning with the highest number in the multiplier, and the highest in their quantity which was to be multiplied. By these means, when multiplying four figures (for example) by four, they produced sixteen different results, which were afterwards to be amalgamated in order to determine the product of the whole. Accustomed as we are from our childhood to the use of the Oriental ciphers, the carrying on the tens seems to be very simple,—and so it is in practice; but the original introduction of this most useful process is wholly artificial, and was of itself by no means originally obvious. The first attempts in endeavouring to grapple with the difficulty, were clearly to secure the largest products, and to correct them by the addition of those which were of smaller value. This has in consequence been found to be the method adopted by self-taught reckoners. In 1827, a lad, of the name of Noakes, was exhibited here in Oxford, who possessed great powers of working sums without paper; and circumstances, which escaped him, led to the supposition of his multiplying from left to right. Upon inquiry this was found to be the case; and such is the force of habit in these operations, that when the contrary order was pointed out to him, he did not seem to appreciate the advantage of it. Another distant and unconnected instance may be brought forward in confirmation of the same view. In January 1712, an account was sent from Dublin to the Royal Society of a boy of 7 or 8 years of age, who worked large sums in his head in this same way; and Dr. Halley remarked at the time, that it was the order in which the Greeks proceeded in their multiplication; their numerals, indeed, were not calculated to suggest the advantages of the contrary method.

The *myriad* was the highest quantity for which there was a specific name in the common language of Greece, and this they went on repeating for the number of times which the occasion might require.

Archimedes saw that by these means the myriad itself became consolidated into a unit, of which 100, 1000, or any multiple might be taken, until the system, as it were, limited itself by reaching a myriad of myriads. He conceived the idea therefore of going on by analogy, and, calling the first his *πρῶτοι ἀριθμοὶ*, to consider the *μύριαι μυριάδες* of them as the monad of his *δευτέροι ἀριθμοὶ*, of which 10, 100, 1000, might again be taken, until they reached in amount to the myriad of myriads which would form the monad of *τρίτοι ἀριθμοὶ*. In this manner, the myriad of myriads of each order became the monad of that which immediately followed it, until this enumeration arrived at the *μυριάκις μυριοστοὶ ἀριθμοὶ*: of these the myriad of myriads might be taken, and here there would be a limit: it would, indeed, require an unit with 800 millions of our ciphers to express the value of it, but still it would be a termination, to provide against which Archimedes denominates all this vast accumulation the *πρῶτα περιόδος* of his system, and consolidates the highest of them into a monad of the *πρῶτοι ἀριθμοὶ* of a *δευτέρα περίοδος*, in which he goes on, till having reached the myriad of myriads of the *μυριάκις μυριοστοὶ ἀριθμοὶ*, the last of them becomes the monad of the *πρῶτοι ἀριθμοὶ τᾶς τρίτας περιόδου*. Thus, he says, we may go on to the completion of the *μυριάκις μυριοστὰ περίοδος*. He proceeds no further in the detail; but it is evident that the last number in this enormous period might, in the same way, be taken for the monad of a new series, and so the system might be continued to any extent.

In modern numeration we have what in a certain degree resembles this plan; for a million of millions makes a billion, a million of billions a trillion, and so on; but these words have not been determined for any very high multiples, and even the billion, the first of them, is hardly ever used, while the rest are so seldom met with, that they hardly belong to common language; and instead of constituting a regularly developed system, they seem rather to be mere examples of the manner in which such names might, if required, be constructed. Now a comparison of the suggestions of Archimedes with what later times have thus adopted, enables us to see the superiority of his mind. The monads in both cases go on in geometrical progression, in which our common ratio would be a million, but his myriad of myriads is 100 millions; so far he has the advantage; but this, though not trifling, is nothing compared with his contrivance for continuing the numeration. In cases of this kind, the attainment of the end depends in great measure on arrangement, and the application of suitable notation and distinctions. Archimedes himself, for the quadrature of the spiral, introduces preliminary propositions, which are founded on the principles by which figures are supposed to be generated in the method of fluxions. Maclaurin[a] has actually made use of them in his valuable treatise on that branch of science, with a particular acknowledgment of the author from whom they were taken; but Archimedes did not generalize the ideas, and adapt expressions to them, by which they might be conveniently brought into extended application. The consequence was, that this most prolific germ re-

[a] Page 59. art. 16.

mained for centuries without its due expansion, and his method of reasoning, which under more favourable circumstances has been the basis of the modern analysis, remained only the noble instance of his mighty individual powers. For the numeration, however, of the high numbers to be used in the Arenarius, he completed the whole system in the most perfect manner; and by assigning general descriptive names to the several parts, he avoided on one side the multitude of arbitrary specific denominations, which would soon have grown overwhelming to the memory; and on the other side he avoided the use of mere symbols, which it would have been impossible to have supplied in sufficient copiousness, after the method of the Greek notation, or to compress into any reasonable space, even now when we have recourse to the Oriental ciphers.

Simplicity and perspicuity were not the only advantages obtained by Archimedes. These large numbers were to be multiplied together, and as they increased, the difficulty of this process might naturally be expected to increase with them; but his arrangement enabled him to point out a method, by which this might be done with as little trouble for the highest orders of his periods as for his lowest numbers. He observed, that beginning from 1, and taking his monads in uninterrupted succession, the multiplying any two of them together would produce a monad, which would lie as far beyond the larger factor, as the smaller was removed from unity[b]. From modern algebra we learn, that when two powers of a quantity are to be multiplied together, the sum of their exponents is equal to that of their product; but this is not all. Archimedes took his geometrical series so as to begin from unity, and if MM be used to denote the myriad of myriads, it will become both the second term and the common ratio; thus MM × MM is the monad of the third order of numbers, MM × MM × MM is the monad of the fourth order, and so on. This series will therefore really be $(\mathrm{MM})^0$, $(\mathrm{MM})^1$, $(\mathrm{MM})^2$, $(\mathrm{MM})^3$, $(\mathrm{MM})^4$, &c. Hence the exponents will go on in arithmetical progression from 0, and at the same time answer respectively to the several terms of a geometrical progression beginning with unity; they thus become so many logarithms, of which the fundamental number, answering to 10 in Briggs's system, is taken as a myriad of myriads. These exponents, however, are in each case one less than the designation of the order of numbers to which the several monads belong; $(\mathrm{MM})^1$ is the monad of the *δευτέροι ἀριθμοὶ*, $(\mathrm{MM})^4$ the monad of the *πεμπτοί*: hence in using them Archimedes refers to the distance of his terms from unity, and not to the order of numbers to which they belong. As his only object was to shew the possibility of enumerating a large quantity, no interpolation was necessary for numbers between the complete powers; and indeed the arithmetic of the time would have made any attempt of that kind unavailing: but still, the principle on which he proceeded is virtually the same which, after the lapse

---

[b] The manner in which he arrives at this conclusion is very curious: *ἔστων ἀριθμοὶ τινὲς ἀνάλογον ἀπὸ μονάδος, οἱ ᾶ, β̅, γ̅, δ̅, ε̅, ζ̅, η̅, θ̅, ι̅, κ̅, λ, μονὰς δὲ ἔστω ὁ α̅· καὶ παραπολλαπλασιάσθω*

ὁ δ̄ τῷ θ̄· ὁ δὲ γενόμενος ἔστω ὁ χ̄—precisely as it would at present be assumed that the quantity to be found was x. He then proceeds regularly in his demonstration, until he shews that *ἴσος ἐστὶν ὁ λ̄ τῷ χ̄*.

of more than 2000 years, led Napier to one of the most valuable and most useful inventions of modern science.

The result of the calculation, carried on in this ingenious manner, was most striking. A vast sphere, far in size exceeding the grasp of man's common apprehension, was to be supposed to be filled with particles so small, that their diameters were less than the $\frac{1}{1100}$ part of an inch. To attempt an enumeration of them had appeared perfectly visionary, and yet Archimedes shews that their quantity would fall short of 1000 myriads of his eighth order of numbers. He was aware that all this, so clearly and so easily expressed, would not carry conviction to the ignorant, but he knew that his conclusion must be admitted by the mathematician. He was conscious not only of the difficulty with which he had had to contend, but of the value of the means which he had devised to overcome it; but he simply concludes by saying *ᾠήθην ὅτι καὶ τινὰς οὐκ ἀναρμοστὸν εἴη ἔτι ἐπιθεωρῆσαι ταῦτα* : but none were then to be found who could follow out his views ;—

> *Οὐ γάρ τις μέτα τοῖος ἀνὴρ ἐν τοίσδεσι πᾶσιν*
> *οἷος ’Οδυσσεὺς ἔσκεν.* Od. xxi. 93.

This sketch of the contents of the *Ψαμμίτης* can give but an imperfect idea of its value; and it is a short tract, which will well repay the time that may be employed in studying it. Hervagius first published it in the edition of the works of Archimedes, which he printed at Basil in 1544. His Greek text is indeed very imperfect, but there is in the same volume a Latin translation, which appears to have been made at an earlier date, and clearly from a much better manuscript than was used for the original language.

Commandine in 1558 gave us another Latin translation, but he appears not to have had a good manuscript for the foundation of his work. Rivaltus, in 1615, published a collection of the works of Archimedes, containing the present tract both in Greek and Latin; but he was confessedly unequal to the work which he had undertaken. Indeed the task was one of considerable difficulty. There is among the earl of Macclesfield's manuscripts, at Shirburn Castle, an original letter, in which Collins mentions that Dr. Barrow, when publishing the other works of Archimedes, at first intended to have omitted the Arenarius on account of the corrupt state of the original text, which he had not time to correct; and there was really no critical edition of this treatise, until that of Wallis was published at Oxford in 1679. It is reprinted in the third volume of his works, with not only his notes and translation, but with the useful addition of notices at the foot of each page, to mark the variations which had been made from the Basil text. It is a valuable work, and Torelli followed it in preparing this part of his edition of Archimedes for the press. Indeed he professes to take it for his standard: he departs from it in very few instances, and those few will be diminished nearly by one half, if we except two of them;—Wallis prints as two words, ἀνὰ λόγον, what Torelli unites into one, ἀνάλογον, and he prints φανηρὸν where Torelli, with all the manuscripts and all the printed editions, reads φανερόν.

In Ballard's collections in the Bodleian, there is an unpublished letter of Dr. Wallis to Dr. Charlett: it has no date, but from the handwriting it is clear that it was drawn up in his old age. "That Archi-

" medes of Syracuse," it says, "did write in the Do-
" ric dialect, we need not question. All that is left
" of his, with which Eutocius hath not meddled,
" remains in the Doric dialect to this day; namely,
" that De conoidibus et sphæroidibus, that De lineis
" spiralibus, his Arenarius, and his Quadratura pa-
" rabolæ......That De sphæra et cylindro, and that
" De dimensione circuli, on which two Eutocius
" hath thought fit to leave us his large commentaries
" for the better understanding of them, are now in
" a dialect very different from the Doric of Archi-
" medes. For Eutocius, it seems, thought fit (for the
" same reasons) to change the text also into the same
" dialect with that of his comment, as more common
" than that of the Doric." Now Wallis, in revising the text, paid great attention to restoring the Doric forms of spelling and accentuation; and the introduction of *φανηρὸν* is certainly systematical and intentional, but no reason is given for it. Maittaire notices this variation; but as he only refers to Wallis's own book as his authority for its existence, he does not supply any additional argument for the alteration[c].

Torelli omits the whole body of Wallis's notes, and merely gives at the foot of his pages some various readings: from these it appears that he had the advantage of a MS. although he has made little or no use of it. He most probably was not aware that another source of elucidation existed in a superior text of the Ψαμμίτης, which had been printed in Great Britain about the middle of the last century. It was never indeed published, and only a limited number of copies got abroad: these likewise were without any title page, or any notice to indicate from whence they came, or under whose care they had been edited. It is probable that the whole history of them would by this time have been lost, if Mr. Barnwell, of the British Museum, had not preserved it, by inserting the particulars in the copy belonging to the library of that establishment. From him we learn, that the book was prepared for the press by Dr. Moor, who was professor of Greek at Glasgow, and who contemplated a complete edition of the works of Archimedes. What may have interfered with the execution of his intentions is not known, but this part was printed in small folio by Foulis with his usual excellence, and the text was taken from a manuscript which was lent to the editor by the Abbé Salier, as it appears, out of the French king's library. When the Oxford edition of Archimedes was published in 1792, various readings were collected from the continent, and added in an appendix: for this purpose collations were procured from four manuscripts in the Royal library of France, [No. 2359, 2360, 2361, 2362,] but none of them answer to Dr. Moor's edition; and Mr. Barnwell tells us, that in 1828, after a diligent search at Paris, he could find no traces of any manuscript which would agree with it[d]. The difference of this from all other editions is very remarkable, its Greek is often much superior to them, but nothing was known till 1830 of any place, in which an original authority was to be found for what had been thus adopted. In the beginning, however, of that year, Mr. Powell of Balliol died, and left his mathematical books to the Oxford Observatory, among which was a MS. of the Ψαμμίτης, in which Mr. Barnwell immediately recognised the readings that distinguished Dr. Moor's edition. A careful collation proves the resemblance to be so strong, that the first impression conveys the idea of it being the Paris MS., but still there may be some room for doubt. Mr. Barnwell mentions the copy of a letter from Dr. Moor which bears immediately upon this point. It is written in French, dated "26 Juillet 1751 N. S." and indicates that it was to be sent by the post to the Abbé Salier, apprising him of Mr. Foulis being about to set out immediately for Paris, when he would restore "le petit MS. du traité d'Archimède." Now we have no knowledge of any impediment to its arrival at the place of destination, and the precaution taken in sending previous notice of its return, would of course occasion inquiries, if it had not been delivered. Mr. Powell was first of Brase Nose, to which his friend Mr. Hindley belonged, by whom the MS. was given to him; but Mr. Hindley unfortunately became lost to

---

[c] Sturzius in his republication of Maittaire's Dialects, makes no addition in this part, (p. 482. C.) Wallis prints *φανερὸν*, in the usual manner, in his edition of Aristarchus Samius &c. No notice is taken (Wallisii Opera III. p. 519, &c.) of *φανηρὸν* as a variation from the Basil orthography, although alterations of much less importance are duly pointed out.

---

[d] It may be remarked likewise that the Greek mathematicians are contained in the second volume of the Cat. Codd. MS. Bibl. Regiæ Par., which was published in 1740 with such great shew of attention to making it complete that an appendix is added at

---

p. 609, "complectens Codd. MSS. Constantinopoli paucis abhinc mensibus in Bibl. Regiam illatos." Montfaucon likewise, in his Bibliotheca Bibliothecarum Manuscriptorum, enumerates only four Greek MSS. of Archimedes in the French king's library, (No. 2153, 4, 5, 6,) which seem to answer to those mentioned above.

society in 1802, and has been dead many years: his friends can give no information of the channel through which the MS. came to his hands, and all is therefore left to conjecture. The book is indeed a small thin volume, but this is not conclusive, since for any short treatise like the present, the transcriber would have hardly been inclined to take a larger page.

The present bishop of Peterborough once endeavoured to devise a formula[d], by which a MS. might be identified from the readings which were found in it; but, as Mr. Porson remarked[e], "a multitude of "considerations are necessary to form a correct "judgment on MSS. which cannot possibly be re- "duced to any theoretical computation," like that on which Dr. Marsh's theorem was founded. In the history of printed books, Ruddiman is said to have published editions of the classics without the deformity of a single press error, and the Glasgow Horace of 1744 has the reputation of being immaculate. Such excellence, however, is the result of repeated revisions; the late Henry Homer never hesitated in cancelling any leaf in which an erratum had been discovered. Now this could hardly have been done, at least to any great extent, in the construction of a MS. Corrections might be and certainly were sometimes introduced, but we are not certain of the care having been taken for other transcripts, which has been so wisely used in examining those of the Hebrew scriptures. But although many copyists must have worked for bread, and, from their haste, have been liable to great inaccuracies, others in monasteries could proceed with more leisure, and may have executed their task with more fidelity. The Oxford MS. is not considered of an earlier date than the 15th century[f], and it is impossible to say that the prototype from which it was taken, or others from the same source, may not now be in existence. None indeed of these are known; but no more was this very book till within these last few years: a number of its best readings coincide with the expressions in the old Latin published by Hervagius;—are we from thence to conclude, that it was the identical MS. from which that translation was made? This could not be maintained, and if it can not, these readings have existed in some other MS., which may yet be brought to light.

---

[d] Letters to Archdeacon Travis, p. 70.

[e] Dr. Milner's Strictures on some of the publications of the Rev. Herbert Marsh, D. D. p. 252.

When an allowance has been made for the restoration of Doric forms and for the corrections of obvious errors, the number of places is very small in which Dr. Moor's text differs from that of our MS. In the copy of his letter to the Abbé Salier, he mentions that a new type was to be formed at Glasgow from that of the Louvre Terence, and that he was waiting for it in order to print a Latin translation of the work. He speaks also of the notes which he intended to annex to the whole. But nothing has been met with to indicate his having executed these parts of his design, and it is impossible to tell what alterations he may have adopted entirely from his own critical conjectures. Wallis, who confessedly had no other means of correcting the text, has suggested many emendations, which are now confirmed by the readings in the Oxford MS. Some allowance, therefore, must be made in reasoning from this similarity. A few discrepancies, which cannot be otherwise accounted for, may have more weight in the opposite scale than a number of instances in which a sameness may be found to exist. In the 4th page of Dr. Moor's edition the words [*διάμετρος ἁλίου*] are printed between brackets. Now these words are neither wanting in the Basil, Rivaltus, or Wallis's editions, nor are they either parenthetical or redundant[g]; it is difficult therefore to imagine why they should have been marked in this manner, unless it were to indicate that they had been inserted in this place without the authority of the MS. from which the text was taken; and they are not deficient in ours. But there is another which seems to be a stronger case: Archimedes, when desirous of assuming some definite measure, which may be less than that of the grains of sand, derives his first quantity from the number of poppy seeds, which, laid beside each other in a straight line, would make up a *δάκτυλος* in length. The Basil Greek makes this 25, but the old Latin and Commandine say 35; Wallis therefore with good reason conjectures that $\overline{\kappa\epsilon}$ has been written by mistake for $\overline{\lambda\epsilon}$, because Archimedes, in his endeavour to take the argument *a fortiori*, immediately says that he shall consider them as not more than 40, which is the next round number above the larger of these quantities. Now Dr. Moor in this place has εἴκοσι ἓξ in words, and he could have no inducement to introduce an alteration of the units; for the argument in favour of λ̄ for κ̄ will be as strong against 26 as against 25. There can be no reason given for what he has printed, but the supposition of his having found it so in his MS., and if he did, it was not the same with ours, which distinctly reads κε̄, or 25, in numerals.

---

[f] It contains an ancient form of the beta *u* (vid. Montfaucon Palæog. Græca p. 336,) but the use of it is confined to the word *ἑβδόμος*. This may be the trace of an old MS. from which it was transcribed. Nothing more can be deduced from it, since, for other words, the common form of the letter is constantly occurring.

---

[g] There is not a single parenthesis introduced in the whole of Dr. Moor's text. One indeed is omitted at p. 16, which stands (δέδεικται γὰρ......τῷ κύκλῳ) in the Oxford MS.

The Quarterly Review[h], in an account of Peyrard's Archimedes, tells us, that "the most valuable "edition of the Arenarius is that published in English at London in 1784, by Mr. George Anderson," and that it "deserves notice, not only as the "first edition of any part of Archimedes' works in a "modern language, but on account of its useful and "ingenious notes and illustrations. It is followed "by a translation of the Latin dissertation of Christopher Clavius on the same subject, and is altogether so valuable, that we cannot but regret that "it has become scarce." It is by no means accurate to describe this as the first translation from Archimedes into a modern language; for the treatise, "De iis quæ humido vehuntur," was published in Italian by Tartaglia in 1551, in French by Pet. Forcadellus in 1565[i], and in English, as a part of his Mathematical Collections, by Salusbury in 1662[k]. J. Ch. Sturm likewise published a German translation of the Arenarius in 1667, and of several of the works of Archimedes in 1670. All this, however, in no way diminishes the value of Anderson's work, which, though it would admit of improvement, is still possessed of considerable merit.

---

[h] Vol. iii. p. 92.

[i] Fabricii Bibliotheca Græca a Harles, vol. iv. p. 177. Petrus Forcadellus was probably Pierre de Forcadel de Beziers.

[k] In the General Dictionary by Bernard, Birch, and Lockman, it is said in a side note (vol. v. p. 372.), with respect to

Anderson entered at Wadham college in March 1780[1], and was appointed Bible clerk in the following July. He was the son of a small farmer of Weston Turvill in the county of Buckingham, and arrested the attention of all who knew him by his early and self-taught acquirements in the mathematics. By the benevolence of the vicar of Whitechurch, near Aylesbury, he was not only sent to college, but was in great measure supported during his residence in Oxford. After proceeding to his bachelor and master's degrees, as soon as he was of standing for them, he took deacon's orders, but appears to have advanced no further, nor to have long directed his thoughts to the pursuits of the profession. His kind friend, the rev. Mr. King,

---

Salusbury's Collections, that " almost the whole impression of " the second tome was destroyed in the fire of London in 1666. " The only copy perhaps now extant is in the curious library of " Will. Jones, esq. F. R. S." Mr. Drinkwater, in his Life of Galileo, (p. 104,) says, that " only one copy is known to be extant in England." He quotes Chauffepié for his authority, probably not being aware that this French writer merely translated in 1750 what had been published in England by Bernard and his coadjutors in 1737. The translation of Archimedes makes a part of the second tome; but having been printed four years previous to the latter part of the Collections, some copies of it got abroad before 1666: one is in the Bodleian. Still, however, they are very scarce, and may easily have escaped the reviewer's notice.

[1] Many of the following particulars are taken from an account of Anderson in the Monthly Magazine, vol. i. p. 345.

afterwards recommended him to Mr. Scrope Bernard, and by the assistance of lord Grenville he obtained employment under government. He is said in the Quarterly Review to have been confidential clerk to the board of control under Mr. Dundas. His real place was "accountant" in that office, and he obtained it about 1787, but he certainly was entrusted by ministers with other confidential business; since his anxiety to complete some calculations for the budget of 1796 made him neglect, till it was too late, a disease which in consequence ended his life within a few days. When this took place he was only in his 36th year; and the president of the board, in a letter of condolence to his widow, described him (apparently with great truth) "as a public loss."

He is said to have made himself well acquainted with the affairs of the East Indies, and to have drawn up a valuable work connected with them; but, with this exception, he seems to have published nothing of any magnitude but the translation from Archimedes. He evidently knew little or nothing of the learned languages before he came to Oxford, and he then applied himself to them rather as a duty than as a source of intellectual gratification; but he was endowed by nature with strong talents, and his proficiency obtained for him one of the Hebrew exhibitions which Dr. Hody had founded in his college. He was enabled to pass the examinations, which he had in consequence to undergo from the regius professor of that language, but still he must have felt that his acquirements in literature were not what he had most to rely upon. His translation of the Arenarius was published about the time of his taking his degree, and, under all the circumstances of the case, does him great credit; but it must be looked upon as the work rather of the mathematician than of the scholar and the critic. He probably had no means of making himself acquainted with some unpublished remarks on the work by Dr. Wallis, which are preserved in the Savilian library; but he speaks[m] of a copy of the Basil Archimedes in the Bodleian, "with MS. corrections by Dr. Bernard," which he could not have examined; for if he had, he would have found that for the Ψαμμίτης they contained little more than a partial insertion of some readings adopted by Rivaltus. These, indeed, could have been of no service to him; but he would have gained a great advantage, if he had not, from an excess of caution, declined to make any use of Dr. Moor's edition, to which nevertheless he frequently alludes. The form in which it got abroad was certainly calculated to excite hesitation, and as he found no information of the authority for the variations in the text, he would have been right to have considered well before he admitted them; but he should have recollected that Wallis's emendations were all conjectural, and even if the later edition had been corrected in the same manner, it became merely a critical question to determine which of the editors had suggested the more probable readings. He explains his reasons in a note[n] on the first page of his book. He points out, that in the very second sentence of the treatise, there is a variation in the old Latin of the Basil edition from the trans-

[m] Page xiv.

[n] Page 33.

lation which he has given; and he goes on to say, “ with this (Latin) the anonymous Greek edition of “ the Arenarius alone agrees, having instead of λέγω, “ ἐγὼ, and adding, to render the sentence complete, “ *πεπερασμένον φαμὶ εἶμεν*. But as there appears no “ other authority for so material an alteration in “ the Greek text and its signification, that meaning “ is here given which seemed to agree best with “ the more ancient Greek copies, and which is also “ given in the Latin translations of Paschasius, “ Commandine, Rivaltus, and Dr. Wallis.” Now the title of the work published by Paschasius Hamellius is only, “ Commentarius in Archimedis li- “ brum de Numero Arenæ;” and although he states the parts of the book as introductions to his several remarks, he does not always give complete and exact translations of them; his expressions, therefore, are only of secondary use in settling the text. Commandine, as has been already mentioned, translated from a bad manuscript, and Wallis says of him [o], that in some places “ sensum potius sequitur “ quam verba.” Of Rivaltus it will not be severe to assert that he can never be depended on; and Dr. Wallis for himself says [p], “ Hervagius, cum vi- “ deatur etiam ea loca, quæ manifesto erant depra- “ vata, ita edidisse ut in manuscripto codice reperta “ fuerant, nobis interim integrum reliquit, per veri “ quæ supersunt vestigia, nostras etiam conjecturas “ interponere, absque *παραδιορθώματος* præjudicio.” His main foundation rested entirely on the imperfect text of Hervagius's edition. As far as authority therefore is concerned, very little from the formidable

[o] Introduction to the notes.

[p] Ibid.

array of Mr. Anderson can be opposed to the better sense which may be found in the Basil Latin. He seems, however, to have unfortunately taken a very mistaken view of this part of the subject. He says[q], that the "anonymous edition of the Arenarius seems "to have been altered on purpose to agree with "the (old Latin) translation." It is now certain that this suspicion was inaccurate; but if it had not been so, still it was no reason for the total rejection of the edition. Its variations from Wallis's are generally for the better, but in many places they go beyond alterations of punctuation, or mere verbal corrections. The adoption of the words in which they are expressed would have required mature deliberation in one who was about to publish the original Greek, but the case was widely different for an English version[r]. It would have been fair in the first instance to presume that such material alterations were not wholly arbitrary; when set, as here stated, at the very lowest estimate, they shewed the editor's confidence in the old Latin translation; and as Mr. Barnwell has well remarked, if Mr. Anderson's objection "were admitted, there still remains the question, from what source did the "translator derive his readings?" His work, as Mr. Anderson himself observes[s], "seems to have "been written long before the publication of the "Greek text;" ("Latinitate jam olim donata, nunc"que primum in lucem edita," are the precise words of its title-page;) it could, therefore, have been only made from an original manuscript, and it is of equal, if not of more authority, than the Greek editions on which Anderson relies. He speaks indeed of "more ancient Greek copies," but all their claim in this respect can only depend upon their having been printed at an earlier period than Dr. Moor's, which is very far from proving that they rested upon more ancient authorities.

---

[q] Page xv.

[r] It may be remarked, likewise, that Mr. Anderson was not very scrupulous in adhering closely to his original, when he thought that he could improve it. Archimedes, after having laid down the absolute quantity in stadia, beyond which he concluded that the diameter of his universe could not extend, goes on to find a magnitude which the grains of sand would not exceed in minuteness. He then, after explaining his system of numeration, proceeds to calculate how many of such magnitudes would be required to make up spheres of different dimensions. Now Mr. Anderson entirely omits the first estimate of the minute grains of sand from its proper place, and interweaves the particulars of the passage with the other second part, in which the calculation begins. It possibly did not strike him, that he was by these means violating the clear method which Archimedes had adopted: and without obtaining any real advantage, he has thus departed from that faithfulness with which a translator ought always to endeavour to exhibit the writings on which he is employed.

To the old translators of the Greek mathematicians, the scientific world is under the greatest obligation. Campanus, in 1482, published Euclid's Elements in Latin from the Arabic, fifty years before that excellent work appeared in its original language; a very curious treatise of Archimedes on floating bodies is preserved to us in Latin, now that the Greek manuscript from which it was translated is no longer known to exist; and the Mathematical Collections of Pappus, one of the most valuable works which issued from the school of Alexandria, are, for the most part, only accessible to the general student in the Latin of Commandine. This last would be admitted as of good authority, if any one undertook to bring out the book in a modern language, and it affords essential service even in correcting the Greek, which is found in the manuscripts. But independent of the advantage which can be derived from such works, (which really are so many perpetual comments on the respective authors,) the case of Barocius's translation of Proclus may be adduced as a strong proof that the Latin sometimes is much more worthy of confidence even than what is found in the original Greek. This likewise is immediately in point; for Hervagius, the same printer who published the Basil Archimedes, published at the same place, in 1533, this Comment of Proclus on the first book of Euclid. The work is disfigured by the speculations of the Pythagoreans and later Platonists; for, as Bacon observes[t] of this very author, "hoc habet ingenium humanum ut cum ad "solida non sufficiat, in supervacaneis se atterat." Still, mixed up with these fanciful notions, many things are recorded of singular curiosity. Among other notices, it preserves an anecdote, the interest of which may be well estimated by its being so universally known. It will likewise give one of the very many instances in which the original Greek is defective. In speaking of Euclid he says, *μέντοι καὶ φασὶν ὅτι Πτολεμαῖος ἤρετο αὐτὸν εἴ τίς ποτὲ ἐστι περὶ γεωμετρίαν.* This broken and enigmatical sentence is all that the original affords; but Barocius, whose Latin translation appeared in 1560, fills up the deficiencies in general, and gives the corresponding passage with complete sense: “Quinetiam ferunt “olim Euclidem a Ptolemæo interrogatum, essetne “aliqua ad geometriam capessendam elementari “institutione brevior via, respondisse nullam esse “viam regiam, quæ ad geometriam ducat.” Now the mutilated Greek text is not the fault of Grynæus, who was the editor of it, but must be attributed to the state of the manuscript from which he printed. In the preface, addressed to Cuthbert Tonstall, he says, “Mihi ipsi Procli commentaria “Oxonii Joan. Claymundus candide suppeditabat.” John Claymond was the first president of Corpus Christi college from 1516 to 1537; and there can be little doubt of his being the person here mentioned, and having supplied the text from a MS. still preserved in the library of that college, in which this passage occurs precisely as it is printed in the Basil edition[u]. Fabricius, in mentioning the Latin of Barocius, says, “Operæ pretium fecisset si “contextum addidisset Græcum,” and it seems extraordinary that he did not; for in his dedication he justly remarks, on the commentaries as printed at Basil, “Adeo laceros et corruptos inveni, ut nihil “boni ex eis elicere potuerim:” but we have his own Greek MS. in the Bodleian, in which we find the authority for his corrections; and from thence we learn the true reading of the sentence just referred to. *Βασιλικὴ ὁδὸς* appears to have been no unusual expression with other writers; and as there has been no second edition of Proclus, it has naturally been adopted whenever allusion is made to this passage; but Euclid told the king in still more terse and definite language, [x] *μηδεμίαν εἶναι βασιλικὴν ἀτραπὸν ἐπὶ γεωμετρίαν.*

---

[s] Page xv.

---

[t] De Augment. Scient. lib. iii. cap. 6.

---

[u] The Corpus MS. also, like the printed book, has no diagrams; and there is another coincidence which is still more remarkable. In p. 24. l. 5. of Grynæus we have *τὸ οὖν ἰσόπλευρον μὴ εἰς ἄπειρον, κ. τ. λ.*; but between *ἰσόπλευρον* and the following word *μὴ*, it appears from the Barocian translation and MS. that there is wanting as much of the Greek text as would possibly occupy a full folio page.—There is exactly the same omission in the same place in the Corpus MS.

---

[x] In the text of the MS. the word is *μ̈η*, which Dr. Gaisford has adopted in his edition of Stobæus, (vol. iv. App. p. 45. note[z],) where he gives the whole passage; but in the margin of the MS., against the same line, we find *μ̈ιαν*, which is probably intended for *μηδεμίαν*, as Barocius translates the word by "nullam."

⁂ Some mistakes having been made with respect to the measurement of the sun's diameter by Archimedes, it may be useful to clear up the obscurity which they have introduced into that part of the treatise.

For the fundamental observation, he says, *τεθέντος μακροῦ κανόνος ἐπὶ ἐπίπεδον, ὀρθὸν ἐν τόπῳ κείμενον ὅθεν ἤμελλεν ἀνατέλλων ὁ ἅλιος ὁρᾶσθαι· καὶ κυλίνδρου μικροῦ τορνευθέντος καὶ τεθέντος ἐπὶ τὸν κανόνα ὀρθοῦ, εὐθέως μετὰ τὰν ἀνατολὰν τοῦ ἁλίου· ἔπειτα, ἐόντος αὐτοῦ ποτὶ τῷ ὁρίζοντι, καὶ δυναμένου ἀντιβλέπεσθαι, ἐπεστράφη ὁ κανὼν ἐς τὸν ἅλιον, καὶ ἁ ὄψις κατεστάθη ἐπὶ τὸ ἄκρον τοῦ κανόνος· ὁ δὲ κύλινδρος, ἐν μέσῳ κείμενος τοῦ τε ἁλίου καὶ τᾶς ὄψιος, ἐπεσκότει τῷ ἁλίῳ· ἀποχωριζομένου οὖν τοῦ κυλίνδρου ἀπὸ τᾶς ὄψιος, ἐν ᾧ τόπῳ ἄρξατο παραφαίνεσθαί τι τοῦ ἁλίου μικρὸν ἐφ' ἑκάτερα τοῦ κυλίνδρου, κατεστάθη ὁ κύλινδρος.*

This text differs but very little from that of Dr. Moor. Hervagius has *ἐπὶ τὸν κανόνα ὀρθοῦ*, but Rivaltus prefers *ὀρθροῦ*, which, as Dr. Wallis remarks, "sensum pervertit." The MS. C, however, of the French king's library, which was collated for Torelli's edition, has *ὀρθροῦ* in the text, and the MS. B has it as a marginal correction; the Oxford MS. has likewise the same reading, and Dr. Moor prints the passage, κυλίνδρου . . . . τεθέντος ἐπὶ τὸν κανόνα, ὀρθροῦ, εὐθέως μετὰ τὰν ἀνατολὰν τοῦ ἁλίου. Now it is difficult to conceive any good reason why the cylinder was to be applied rather at this particular time than at any other previous to the observation. This applies to both the readings; but there is a contradiction in speaking of the “twilight, immediately after (or even at) the rising of the sun.” There can, therefore, be no doubt of this reading being wrong, and that the original word restored by Dr. Wallis is right.

The κανὼν was a straight rod, which having been placed flat against the side of an upright plane, a well turned cylinder was contrived to move along it at right angles to that plane: this was placed at first so near the eye as entirely to cover the sun's disc, and then gradually removed till a small quantity of the sun's light became visible on each side of it.

Paschasius Hamellius saw accurately from this description, that Archimedes observed the perpendicular, and not the horizontal diameter of the sun. It was the more easy to observe, and Archimedes might not have been aware of the error which was thus occasioned by the difference of refraction on the upper and lower limbs; or having marked the oval form of the orb on the horizon, he may have taken the shorter diameter in order to give the greater force to his argument. His object was to deduce the sun's distance from its assumed magnitude and apparent diameter; consequently the less the observed angle, the greater would be the radius of his κόσμος.

Hervagius begins the description by τεθέντος μακροῦ κανόνος ἐπίπεδον, and the old Latin translation renders it “posita longa regula super planum erectum;” but Rivaltus, finding nothing to govern ἐπίπεδον, proposes to divide the word into two, ἐπὶ πέδον, and confuses the whole by supposing that the cylinder lay flat on an horizontal table, which was placed upon an upright stand. Dr. Wallis saw that this position was objectionable, and incompatible with the reading which he adopted; but the deficiency of grammatical construction induced him to admit the conjecture of ἐπὶ πέδον, in which he was certainly mistaken. “Super fulcrum” is the sense which he gives to the words; but πέδον is a distinct root, not a derivative from ποῦς; it means the ground itself, not a support, which may raise any thing above the surface. He says also, “erigendus cylindrus est ad regulam normalis,” which seems to indicate that he considered the cylinder to be used for measuring the horizontal diameter of the sun, and this appears to be the sense in which Mr. Anderson has understood the passage, but there can be no doubt that all difficulty and ambiguity are best removed by Dr. Moor's reading, which agrees with that of the Oxford MS. Nothing can be more easy to conceive than that the transcriber overlooked the first in the consecutive repetition of ἐπί.

In a subsequent demonstration Archimedes takes the case when the sun has risen above the horizon, and argues from its centre being then further from that of the earth than from the place of observation. Nothing can be more clear and simple than this truth, as derived from the diagram commonly used for the illustration of parallax in altitude; but Dr. Wallis has thought it necessary to add an explanation, in which he deduces this fact from the obtuse angle which will be made by the line drawn from the eye of the observer to the centres of the sun and the earth, which he says must be so "propter solis diametrum majorem quam telluris." Now the annexed reason only creates a difficulty. It is hard at first to see what difference can be supposed to exist in this respect between the sun's centre and a fixed star; but the error is derived from Commandine's commentary, where he introduces the case of the sun being on the rational horizon, and says, that as its semidiameter is greater than that of the earth, its centre would then be raised above the sensible horizon. Now there is no reason whatever for this refinement. Archimedes argues from the angle which he observed the perpendicular diameter of the sun to subtend, and even if any doubt could remain of his meaning, it must be removed by his precisely representing the position of the horizon, in his diagram, by a line which is a tangent to the earth's surface at the point where the observation is supposed to be made.